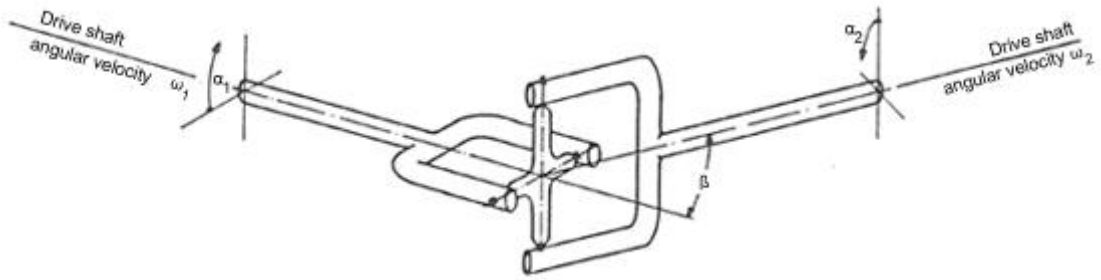
Kinematik of Universal Joints

If two shafts bent at angles to each other are connected with an universal joint (Fig. 9) and the drive shaft moves at constant angular velocity ω_1, the driven shaft runs with uneven angular velocity ω_2, i.e. the twist angle α_2 of the driven shaft does not at any point agree with the twist angle α_1 of the drive shaft. The difference angle $\Delta\alpha$ and hence the degree of unevenness u depends on the deflection angle of the joint.

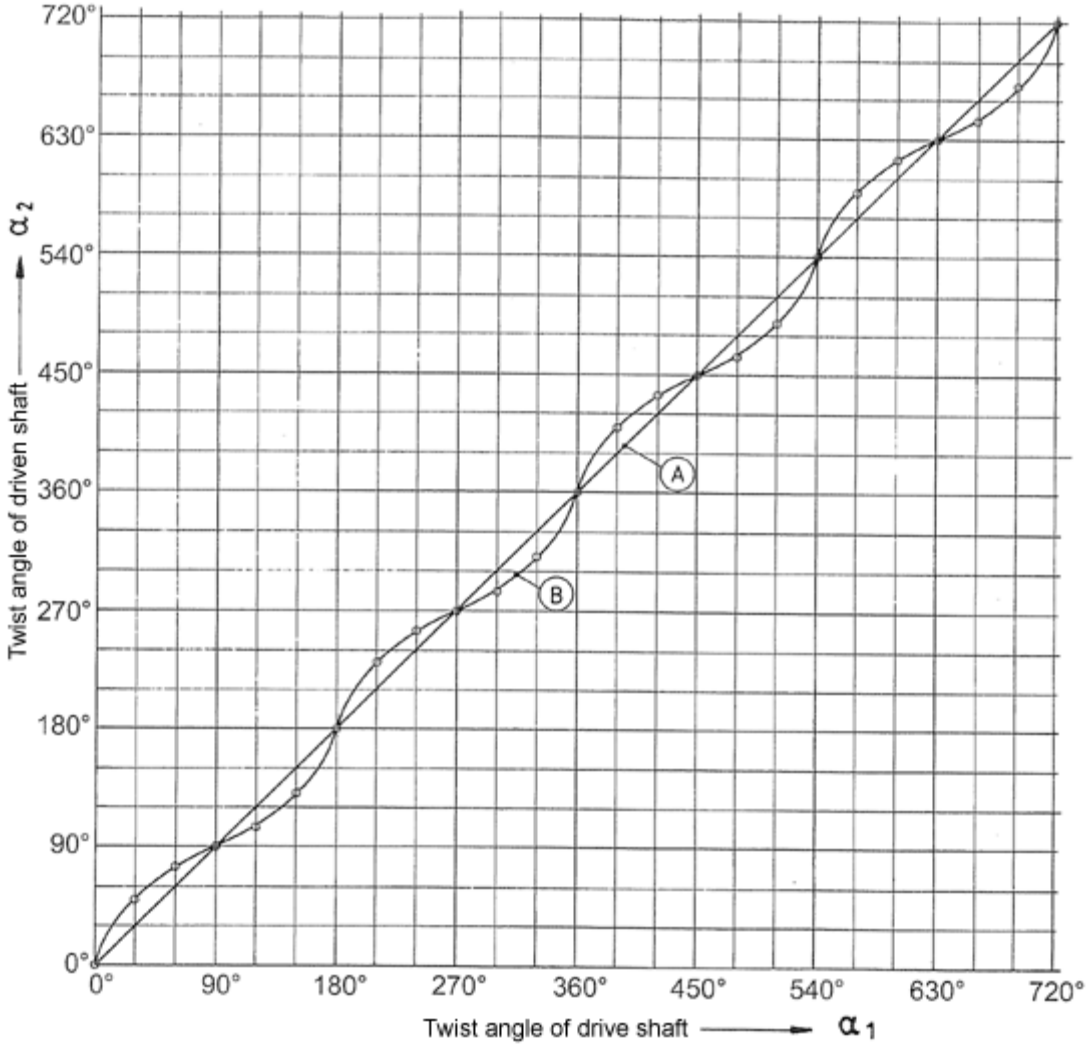
Fig. 9:



The correlations are shown in the following figures 10 to 13.

Fig. 10:

$$\alpha_2 = \arctan \frac{\tan \alpha_1}{\cos \beta}$$

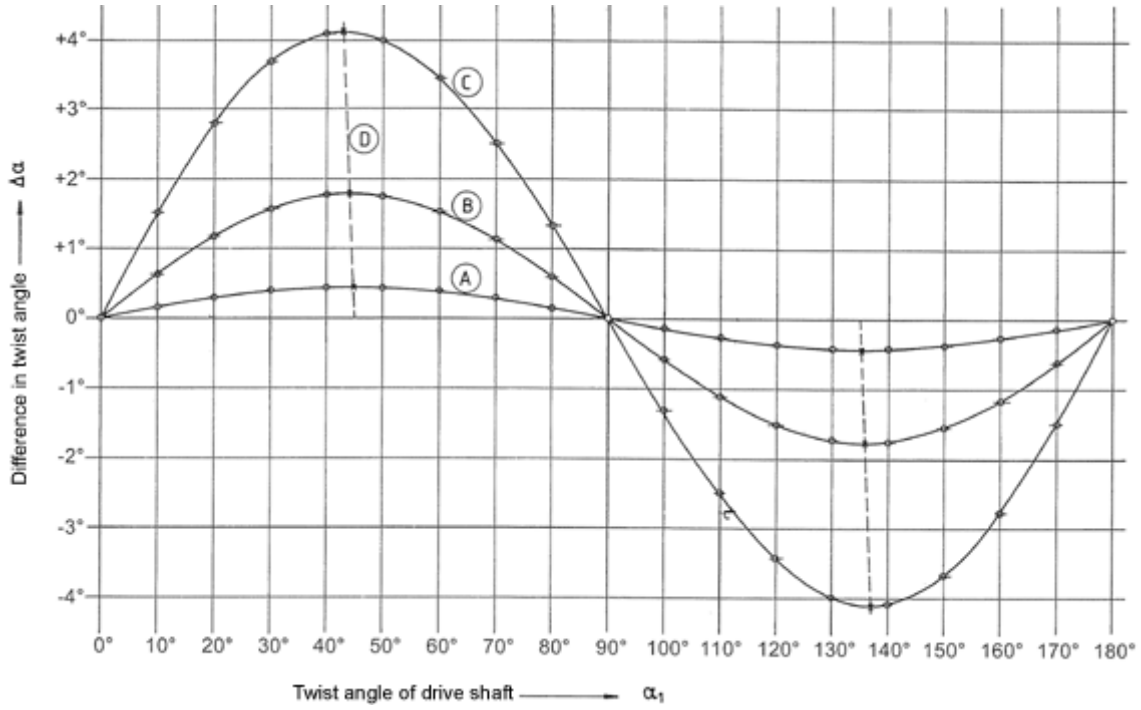


Curve A: $\alpha_2 = \alpha_1$ (synchronous line)
Curve B: $\alpha_2 = f(\alpha_1; ß \text{ odd } 0°)$

Difference in twist angle (cardan fault)

$$\Delta \alpha = \alpha_2 - \alpha_1$$

Fig. 11:

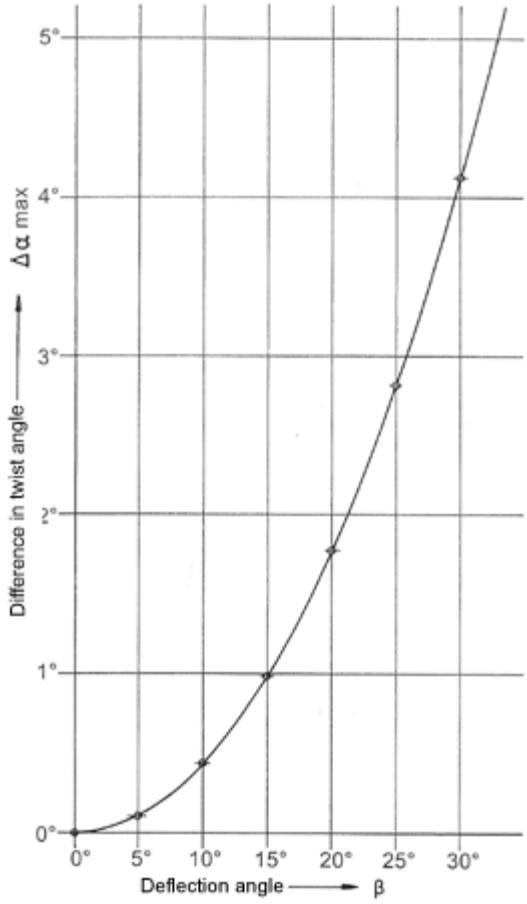


Curve A: Deflection angle $\beta = 10°$, $\Delta\alpha_{max} = \pm\ 0{,}438°$ at $\alpha_1 = 44{,}8°$
Curve B: Deflection angle $\beta = 20°$, $\Delta\alpha_{max} = \pm\ 1{,}782°$ at $\alpha_1 = 44{,}1°$
Curve C: Deflection angle $\beta = 30°$, $\Delta\alpha_{max} = \pm\ 4{,}117°$ at $\alpha_1 = 42{,}9°$
Curve D: Stationary line of $\Delta\alpha_{max}$

Max. difference in twist angle

$$\Delta\alpha_{max} = \arctan \frac{1-\cos\beta}{2\sqrt{\cos\beta}}$$

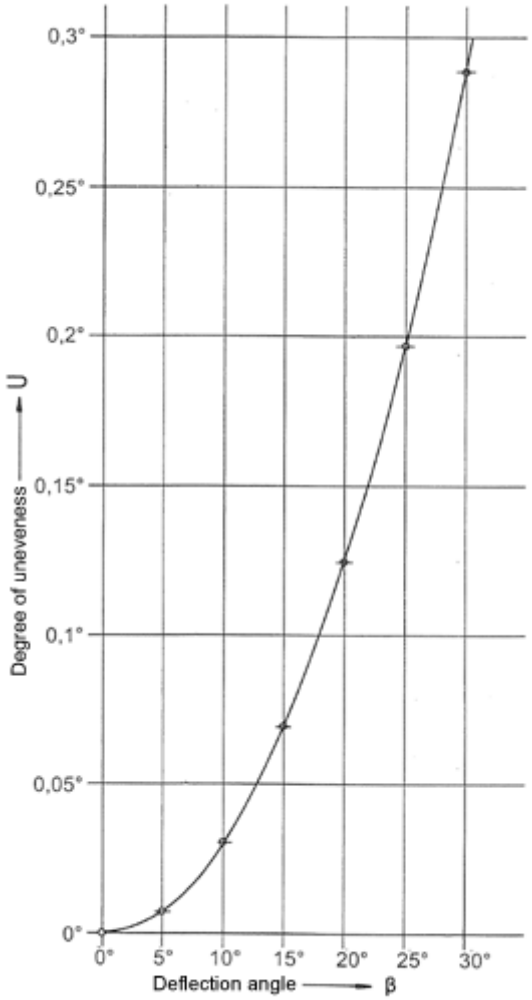
Fig. 12:



Degree of unevenness

$$u = \frac{\omega_{2max} - \omega_{2min}}{\omega_1} = \tan\beta \cdot \sin\beta$$

Fig. 13:


0,3°
0,25°
0,2°
0,15°
0,1°
0,05°
0°
Degree of uneveness U
0°
5°
10°
15°
20°
25°
30°
Deflection angle β